# Capacity Controlled Demand Side Management: A Stochastic Pricing Analysis

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***Abstract*—We consider a novel paradigm for demand side management, assuming that an aggregator communicates with a household only at the meter, imposing a capacity constraint, i.e. a restriction on the total power consumption level within a given time frame. Consumers are then responsible to adjust the set-points of the individual household devices accordingly to meet the imposed constraint. We formulate the problem as a stochastic household energy management program, with stochasticity arising due to local photovoltaic generation. We show how a demand bidding curve for capacity increments can be constructed as a by-product of the developed problem and provide a rigorous pricing analysis that results in a probabilistic "shadow" price envelope. To evaluate the efficacy of the proposed approach we compare it with an idealized real-time market price set-up and show how our analysis can provide guidelines to consumers when selecting a service contract for load curtailment.**

***Index Terms*—Demand side management, aggregated demand response, pricing, duality theory, stochastic optimization.**

## NOMENCLATURE

| | |
|---|---|
| $N_L$ | Number of uncontrollable loads. |
| $N_c$ | Number of controllable loads. |
| $N_{PV}$ | Number of photovoltaic (PV) generators. |
| $N_s$ | Number of storage devices. |
| $N_f$ | Number of grid points for the discretized capacity profile. |
| $N$ | Optimization horizon. |
| $T$ | Granularity of capacity profile. |
| $P_L^j(k)$ | Power of uncontrollable load $j = 1, \dots, N_L$ at time $k = 1, \dots, N$. |
| $P_c^j(k)$ | Power of controllable load $j = 1, \dots, N_c$ at time $k = 1, \dots, N$. |
| $P_{c,\text{base}}^j(k)$ | Baseline power of controllable load $j = 1, \dots, N_c$ at time $k = 1, \dots, N$. |
| $P_{PV}^j(k)$ | Power of PV generator $j = 1, \dots, N_{PV}$ at time $k = 1, \dots, N$. |
| $P_s^j(k)$ | Power exchanged with storage device $j = 1, \dots, N_s$ at time $k = 1, \dots, N$. |
| $x^j(k)$ | Energy content of storage device $j = 1, \dots, N_s$ at time $k = 1, \dots, N$. |
| $x_\ell^j$ | Stored energy degradation related to device $j = 1, \dots, N_s$. |
| $\underline{x}^j, \overline{x}^j$ | Minimum and maximum storage limits for device $j = 1, \dots, N_s$. |
| $\eta^j$ | Charging/discharging efficiency of storage device $j = 1, \dots, N_s$. |
| $P_f(i)$ | Capacity limit at time $i = 1, \dots, N/T$. |
| $\bar{P}_f$ | Discretized capacity profile including $N_f$ values of $P_f(i)$. |
| $\delta^j(k)$ | Forecast error related to PV generator $j = 1, \dots, N_{PV}$ at time $k = 1, \dots, N$. |
| $d_+^j(k), d_-^j(k)$ | Positive and negative forecast error allocation coefficients for $j = 1, \dots, N_c$, $k = 1, \dots, N$. |
| $P^j(k, \delta)$ | Power dispatch policy for load $j = 1, \dots, N_s$ at time $k = 1, \dots, N$, contingent on $\delta$. |
| $U^j(k, \delta)$ | Disutility of load $j = 1, \dots, N_c$ at time $k = 1, \dots, N$ contingent on $\delta$. |
| $\rho^j(k)$ | Baseline power deviation penalty coefficient for $j = 1, \dots, N_c$, $k = 1, \dots, N$. |
| $\mathbb{P}$ | Probability measure with support $\Delta$. |
| $\mathcal{R}_{\delta \in \Delta}[\cdot]$ | Risk metric. |
| $m$ | Number of PV forecast error scenarios extracted according to $\mathbb{P}$. |
| $S_m$ | Discrete set including $m$ PV forecast error scenarios. |
| $\lambda[S_m, P_f(i)]$ | "Shadow" price associated to scenario set $S_m$ and capacity limit $P_f(i)$. |
| $\bar{\lambda}[S_m, \bar{P}_f(i)]$ | Average "shadow" price associated to scenario set $S_m$ and capacity limit $\bar{P}_f(i)$. |
| $\epsilon$ | Violation level taking values in $[0, 1]$. |
| $\beta$ | Confidence level taking values in $[0, 1]$. |
| $M$ | Number of service contracts. |
| $c_j$ | Purchase price ($/KW/year) related to contract $j = 1, \dots, M$. |
| $p_j$ | Probability of curtailment (hours/year) related to contract $j = 1, \dots, M$. |

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## I. INTRODUCTION

POWER systems are one of the most critical infrastructures in modern society. To ensure reliable system operation, control services of a different nature need to be provided. This task has become more challenging due to the increased level of uncertainty as a result of the increasing penetration of renewable energy sources. To account for this uncertainty not only conventional scheduling and regulation problems need to be revisited, but also conceptually different modeling and control schemes have to be designed.

The conventional approaches involve mainly generation side control. This control method requires adjusting the output of the generators and includes various operational challenges which span different time scales [1]. To account for the intermittent nature of the renewable generation, as well as for other uncertainty sources in the system, research has focused on formulating the stochastic counterparts of the aforementioned problems. Representative work in this context, including stochastic reserve scheduling and unit-commitment under security and/or market constraints, can be found in [2–6].

An alternative approach, in a sense dual to generation side control, is the so called load side control or demand side management. While controlled loads offer an additional degree of freedom when solving regulation or planning problems in the presence of uncertainty, they can also provide a reliable resource to the power network without any disruption of service to the consumers [7], [8]. Toward this direction different approaches for demand side management have been proposed in the literature. Following [8], we can distinguish between price and direct load control. The first approach is based on providing real-time price signals to consumers [9], which will then respond to those signals by appropriately adjusting their power consumption level. Such a set-up, however, imposes challenges related to power system stability [10]. Direct load control offers an alternative approach for demand side management. Direct control of devices and appliances in the household such as thermostatically controlled loads, electric vehicles, water heaters, etc, at an individual or population basis, has attracted significant attention in the literature [11–18].

Another paradigm, that falls between price and direct load control, is discussed in [19]. In this framework an aggregator communicates with a household or a residential area only at the meter by imposing a capacity constraint, i.e. a restriction on the aggregated power consumption level within a specific time frame. Consumers are then free to satisfy this constraint by appropriately adjusting the set-points of the individual household devices as they choose. This approach is less intrusive than direct load control since it enables consumers to meet their contract obligation in many ways that reflect changes in valuation, and does not raise stability issues as in price based control [10]. A conceptually similar work, but following a completely different formulation, is presented in [20]. In that paper the authors provide necessary and sufficient conditions for a supply profile to be adequate for meeting an energy requirement for an aggregation of consumers. This energy requirement parallels the capacity constraint we consider in the this paper. Our work is also complimentary to [21], where the author considers capacity constraints from a mechanism design point of view and motivated by priority of service pricing advancements [22–25], proposes a capacity subscription mechanism to deal with the problem of matching supply and peak demand.

In this paper we focus on the capacity control paradigm and formulate the household energy management problem as a disutility minimization stochastic optimization program subject to the capacity constraint, with stochasticity arising due to local photovoltaic (PV) power generation (load uncertainty can be included similarly). To simplify our analysis we first present our results without including storage devices. We then show in Appendix A how our framework can be extended to include storage dynamics as well. Household energy management problems or mathematically similar formulations have attracted significant attention in the literature [26–28]. In the same context and in the presence of a budget constraint that is similar to the capacity constraint considered here, [29] propose a Stackelberg game to deal with the utility revenue and end-user pay-off maximization problem.

We build on such models and extend them appropriately so that they are included in a scenario based stochastic optimization set-up, where the computed optimal solution is accompanied with an a-priori probabilistic certificate regarding the satisfaction of the problem constraints. Moreover, instead of using only open loop decisions (first-stage variables), we introduce recourse functions of the uncertainty (second-stage variables), that offer additional degrees of freedom, leading to more optimal solutions in terms of cost. We also show that a demand curve for capacity increments, which consumers can reveal to the aggregator explicitly or through contract selection, can be constructed as a by-product of the developed problem. This demand curve can then be used by the aggragator to create demand side offers. We provide a rigorous analysis and construct an envelope around the demand curve related to the deterministic variant of the proposed problem that encloses the demand curve corresponding to the stochastic problem. To quantify the disutility due to load curtailment when using the constructed curve for bidding in the market, we compare our approach with a real-time market price set-up. We also show how the proposed scheme can be used to provide guidelines when selecting a service contract for load curtailment.

Section II formulates the household stochastic optimization program arising under the proposed capacity controlled demand side management paradigm. In Section III we show how a demand curve for capacity increments can be constructed and provide a pricing analysis in a stochastic set-up. Section IV includes a simulation based analysis, whereas Section V concludes the paper and provides directions for future work. Appendix A shows how the developed framework can be extended to include storage devices, whereas all proofs can be found in Appendix B.

## II. Capacity controlled demand side management

### A. Problem statement

Consider a household comprising $N_L$ uncontrollable loads, $N_c$ controllable loads that will be used to provide demand response services and $N_{PV}$ photovoltaic (PV) generators. Note that in a single household it is more likely to have only one PV generator, i.e. $N_{PV} = 1$. However, we consider here multiple PV units to allow for a more general formulation that is not necessarily limited at a household level, but is applicable to the load management problem of an entire residential area. Following [26] we show in Appendix A how to extend our framework to include storage devices.

We consider a set-up where an aggregator interacts with the household only at the household meter. Specifically, the household is subscribed to a capacity limit, that is activated remotely by the aggregator. In general, the capacity limit may be contingent on some exogenous state variable defined in the service contract. Once this capacity limit is imposed, the household is responsible to optimize the schedule of the individual devices in the most cost efficient way by adjusting their set-points, while the aggregator is not involved in this process. These set-points will then be tracked by some low level controller with which we assume each device is equipped. Therefore, when computing the load set-points, the particular consumer type does not need to be known. For more details on possible implementations of this paradigm we refer to [19].

Let $N$ denote an optimization horizon with unitary steps. Every $T$ steps a capacity limit is imposed, representing a budget constraint that requires the total net load in the household not to exceed this limit. Assume that $N$, $T$ are such that $N/T$ is an integer, and let $\{P_f(i)\}_{i=1}^{N/T} \in \mathbb{R}^{N/T}$ be a vector that includes the capacity limits and will be referred to as capacity profile for the rest of the paper. Moreover, for each $k = 1, \dots, N$, $j = 1, \dots, N_L$, $P_L^j(k) \in \mathbb{R}$ denotes the power of the uncontrollable load $j$ in time-step $k$, and is treated as a constant in our analysis. Similarly, for $k = 1, \dots, N$, $j = 1, \dots, N_c$, $P_c^j(k) \in \mathbb{R}$ denotes the power of the controllable load $j$ in time-step $k$.

For $k = 1, \dots, N$, $j = 1, \dots, N_{PV}$, let $P_{PV}^j(k)$ represent the PV power forecast of generator $j$ in time-step $k$. Since forecasts are in general inaccurate, we will perform a stochastic analysis, taking forecast errors into account. For each $j = 1, \dots, N_{PV}$, let $\delta^j = (\delta^j(1), \dots, \delta^j(N))$ be a vector including the forecast error of each PV unit. Moreover, let $\delta = (\delta^1, \dots, \delta^{N_{PV}}) \in \Delta$ be distributed according to an absolutely continuous distribution $\mathbb{P}$ with compact support $\Delta$. This distribution may be unknown, but we assume that we are able to extract, or we are provided with, samples from this distribution (e.g. historical data). The continuity assumption is only needed in the proof of Theorem 1. Since all forecast errors for the individual units and the different time-steps, are collectively included in $\delta$, spatial and temporal correlation is respected once a sample is extracted from $\Delta$ according to $\mathbb{P}$.

We treat the household response to the imposed capacity limits as a disutility minimization problem, where the objective is to find the optimal dispatch for the household loads that minimizes the deviation from a baseline load profile, which is assumed to be fixed (e.g. it may correspond to the solution of the deterministic variant of the problem). Specifically, we seek to determine a load dispatch policy that minimizes $\sum_{k=1}^{N} \sum_{j=1}^{N_c} \mathcal{R}_{\delta \in \Delta}[U^j(k,\delta)]$, where $\mathcal{R}_{\delta \in \Delta}[\cdot]$ is any given risk metric. For example, it can represent the expected value of its argument or its worst case value (take $\mathcal{R}_{\delta \in \Delta}[\cdot] = \sup_{\delta \in \Delta} || \cdot ||$, where $|| \cdot ||$ is the first or the Euclidean norm). For $k = 1, \dots, N$, $U^j(k,\delta) \in \mathbb{R}$ denotes the disutility of load $j = 1, \dots, N_c$. We consider $U^j(k,\delta) = \rho^j(k)\big(P_{c,\mathrm{base}}^j(k) - P^j(k,\delta)\big)$, to penalize the deviation of the load dispatch policy $P^j(k,\delta)$, whose structure will be defined next, from a baseline load level $P_{c,\mathrm{base}}^j(k) \in \mathbb{R}$, i.e. we assign a penalty to load curtailment. Coefficient $\rho^j(k) \in \mathbb{R}_+$ is a penalty factor, possibly different for each $j = 1, \dots, N_c$, $k = 1, \dots, N$. Notice that, even not shown explicitly, $U^j(k,\delta)$ depends on the decision variables $P_c^j(k), d_+^j(k), d_-^j(k)$ that constitute the load dispatch policy $P^j(k,\delta)$ and will be defined in the sequel.

We thus have the following family of problems, parameterized by the uncertainty set and the capacity profile, and we will refer to each of them as $\mathcal{P}[\Delta, \{P_f(i)\}_{i=1}^{N/T}]$.

$$\min_{\left\{\left\{P_c^j(k),\ d_+^j(k),\ d_-^j(k)\right\}_{j=1}^{N_c}\right\}_{k=1}^{N}} \sum_{k=1}^{N} \sum_{j=1}^{N_c} \mathcal{R}_{\delta \in \Delta}[U^j(k,\delta)] \quad (1)$$

subject to:

1) Capacity constraint: For each $i = 1, \dots, N/T$ the total net load in the household should be restricted to the corresponding element of the capacity profile, for all $\delta \in \Delta$, i.e.

$$\sum_{k=iT-T+1}^{iT} \Big[ \sum_{j=1}^{N_L} P_L^j(k) - \sum_{j=1}^{N_{PV}} \big(P_{PV}^j(k) + \delta^j(k)\big) + \sum_{j=1}^{N_c} P^j(k,\delta) \Big] \leq P_f(i),\ \forall \delta \in \Delta, \quad (2)$$

where $P^j(k,\delta) \in \mathbb{R}$ is the sum of a deterministic component which is the dispatch $P_c^j(k)$ of the controllable loads, and two terms that depend on the uncertain error and are mutually exclusive. Specifically, we assume a piecewise affine control rule, represented by

$$P^j(k,\delta) = P_c^j(k) + d_+^j(k) \max\big(0, \sum_{\ell=1}^{N_{PV}} \delta^\ell(k)\big) - d_-^j(k) \max\big(0, -\sum_{\ell=1}^{N_{PV}} \delta^\ell(k)\big). \quad (3)$$

Note that $P_c^j(k)$ can be thought of as a first stage decision, whereas $d_+^j(k),\ d_-^j(k) \in \mathbb{R}$ can be thought of as the coefficients of an affine recourse action. In particular, the stochastic terms imply that if an uncertain error is realized, it should be allocated to the controllable loads according to the coefficients $d_+^j(k),\ d_-^j(k)$, adjusting their set-point $P_c^j(k)$. If the total forecast error is positive, the loads should increase their power consumption, while if it is negative they should decrease it. To encode this error allocation protocol we impose the following set of constraints on the coefficients $d_+^j(k),\ d_-^j(k)$.

Despite the structural similarities, (2) is a power and not an energy constraint, imposing a limit on the the sum of the net household power consumption taken over $T$ consecutive time steps, thus offering additional flexibility in satisfying the constraint compared to a limit on the instantaneous power. Taking $T = 1$, (2) reduces to a limit on the net power consumption at every time step.

2) Allocation constraints: For each $k = 1, \dots, N$, the allocation coefficients should satisfy

$$\sum_{j=1}^{N_c} d_+^j(k) = 1,\ \sum_{j=1}^{N_c} d_-^j(k) = 1,$$
$$d_+^j(k), d_-^j(k) \geq 0,\ \forall j = 1, \dots, N_c, \quad (4)$$

which imply that they should be positive and sum up to one. The positivity of the allocation coefficients is required only in the proof of Proposition 2. If constructing a "shadow" price envelope is not desirable we could allow the allocation coefficients to be also negative, since this may lead to more profitable solutions for some choices of the objective function.

3) Controllable load limits: For each $k = 1, \dots, N$, $j = 1, \dots, N_c$, the set-point of each load together with its adjustment in case of a forecast error should satisfy

$$\alpha^j(k) P^j_{c,\text{base}}(k) \leq P^j(k, \delta) \leq P^j_{c,\text{base}}(k),\ \forall \delta \in \Delta, \quad (5)$$

where $P^j(k,\delta)$ is given by (3) and $\alpha^j(k) \in [0,1]$ characterizes the flexibility margins of each load. Note that we only allow loads to be curtailed compared to the baseline profile; flexibility in increasing load consumption from the baseline level can be modeled analogously.

Problem $\mathcal{P}[\Delta, \{P_f(i)\}_{i=1}^{N/T}]$, given by (1)-(5), is a constrained minimization problem. Since we do not have coupling constraints between consecutive $T$ time-steps, we could consider $N = T$ and solve the optimization problem for every $T$ steps in parallel. This can also simplify the averaging procedure of Section III-A; this is not the case, however, if we include storage dynamics as in Appendix A.

Note that loads are typically fluctuating, therefore, for a more realistic model we could represent them by stochastic processes, i.e. superimpose to $P^j_L(k)$ a stochastic term. This could be easily included in the proposed framework. Given any stochastic time-series model for the load consumption or availability of historical data, we could treat load stochasticity in a similar manner as stochasticity in PV power and generate scenarios (see Section II-B) for the load uncertainty error as well. However, it should be noted that since the uncertainty error due to load fluctuations is typically smaller that the error in PV power production, it is reasonable to assume that the former is dominated by the latter, thus justifying our choice to represent $P^j_L(k)$ as a deterministic quantity.

It should be also noted that in practice loads change their consumption in discrete amounts and can not be controlled continuously. This is in contrast with our formulation where, following [9], [14], [20], [26], we assume load curtailment to be a continuous action. To implement the resulting dispatch policy a "closest neighbor" approximation could be employed, dispatching each load to the value among the discrete set of admissible dispatches that is closer to the solution of the proposed program that involves continuous decisions. The more loads are included in the aggregation, the less frequent it will be for such a scheme to result in feasibility or optimality issues, since the approximation errors would average out among the different loads.

### B. Problem reformulation

By inspection of (2), (4), it can be shown that $\mathcal{P}[\Delta, \{P_f(i)\}_{i=1}^{N/T}]$ is equivalent to a problem that involves minimizing the same objective function (1) subject to (3), (4), (5), but with the uncertain capacity constraints being replaced by deterministic constraints.

**Proposition 1.** *Problem $\mathcal{P}[\Delta, \{P_f(i)\}_{i=1}^{N/T}]$ is equivalent to a problem that involves minimizing* (1) *subject to* (3), (4), (5) *and*

$$\sum_{k=iT-T+1}^{iT} \Big[ \sum_{j=1}^{N_L} P^j_L(k) - \sum_{j=1}^{N_{PV}} P^j_{PV}(k) + \sum_{j=1}^{N_c} P^j_c(k) \Big] \leq P_f(i). \quad (6)$$

Note that the introduction of the allocation coefficients, which are crucial for the proof of Proposition 1, is inspired by the analysis of [6], where such coefficients were introduced to allocate the generation-load mismatch among the various generating units. Proposition 1 parallels the fact that in [6] only a deterministic power balance constraint has to be imposed.

Problem $\mathcal{P}[\Delta, \{P_f(i)\}_{i=1}^{N/T}]$ is then given by (1)-(5), with (2) replaced by (6). However, (5) should be satisfied for all $\delta \in \Delta$. $\Delta$ may be an infinite set, rendering $\mathcal{P}[\Delta, \{P_f(i)\}_{i=1}^{N/T}]$ a semi-infinite optimization program, which is not easy to solve in general. Therefore, we relax (5) and impose the load limit constraints not for every $\delta \in \Delta$, but for any $\delta \in S_m = \{\delta_i\}_{i=1}^m$, where $S_m \subset \Delta$ is a discrete set containing $m$ identically and independently distributed realizations of the uncertain error. This gives rise to a linear program with constraints that should be satisfied only for a finite number of uncertainty scenarios. Due to the decoupled structure of the problem, for any $k = 1, \dots, N$, it suffices to enforce (5) only for the extreme values of $\sum_{\ell=1}^{N_{PV}} \delta^\ell(k)$, among the samples in $S_m$. Moreover, the risk metric is substituted by $\mathcal{R}_{\delta \in S_m}[\cdot]$.

The resulting family of optimization programs can be denoted as $\mathcal{P}[S_m, \{P_f(i)\}_{i=1}^{N/T}]$. We assume throughout the paper that $\mathcal{P}[S_m, \{P_f(i)\}_{i=1}^{N/T}]$ is feasible, its feasibility region has a non-empty interior and it admits a unique optimal solution. Fix $\epsilon,\ \beta \in (0,1)$ and extract $m \geq \frac{e}{e-1}\frac{1}{\epsilon}\big(N_{PV}N - 1 + \ln\frac{1}{\beta}\big)$ samples to construct $S_m$. Following [30], with confidence at least $1-\beta$, the minimizer of $\mathcal{P}[S_m, \{P_f(i)\}_{i=1}^{N/T}]$ satisfies (3)-(6) with probability at least $1-\epsilon$. This implies that we can accompany our solution with an a-priori (probabilistic) certificate regarding the satisfaction of the system constraints.

The sample size bound scales logarithmically with respect to $\beta$, which implies that we can select $\beta$ very close to zero ($\beta = 10^{-11}$ in the example of Section IV) without an unaffordable increase in the number of samples $m$ that need to be extracted. Therefore, the feasibility statement can be provided with confidence close to one. Note that the number of samples that need to be extracted for the aforementioned probabilistic feasibility statement to hold, depends linearly on the total number of uncertain variables $N_{PV}N$ due to [30]. Other sample size bounds can be used as well [31], [32].

## III. Pricing analysis

### A. Demand curve for capacity increments

In Section II-B we formulated a family of problems $\mathcal{P}[S_m, \{P_f(i)\}_{i=1}^{N/T}]$ parameterized by $S_m$ and $\{P_f(i)\}_{i=1}^{N/T}$. For any given capacity profile, $\mathcal{P}[S_m, \{P_f(i)\}_{i=1}^{N/T}]$ provides the load dispatch that minimizes the total load disutility. For each $i = 1, \dots, N/T$, the dual variable associated with each constraint in (2) shows the effect in the disutility of an incremental change in $P_f(i)$. Let $\lambda[S_m, P_f(i)] \in \mathbb{R}_+$ denote this dual variable. Variable $\lambda[S_m, P_f(i)]$ should be interpreted as a "shadow" price. This intuitive interpretation is due to the fact that we have uncertainty independent capacity constraints, as an effect of the use of the allocation vectors. This is in contrast to other stochastic scheduling approaches that introduce a different set of decision variables (increasing on the same time the computational burden) and enforce different constraints in the form of (6) per uncertainty sample. This results in a different dual variable per sample and it is then unclear which of them (or their expected value) should be selected as "shadow" price.

In the proposed framework we assume that participation of retail customers in the day-ahead and real-time market is handled through a third party aggregator or through the retail supplier by offering customers load control contracts that allow the aggregator or retail supplier to bundle contracted load control options at the retail level into wholesale demand response products that can be offered into the wholesale markets as day-ahead energy, real-time energy or ancillary service products. We do not consider injection by consumers into the grid and local PV power is only assumed to meet local consumption. Under this set-up we aim at constructing a demand curve that will then be revealed to the aggregator. The revelation of the demand curve to the aggregator is implicit through a mechanism that invokes the revelation principle [33]. The mechanism design is outside the scope of this paper but the core idea is that the demand curve will be revealed by the consumer through the selection of a contract for each load increment out of a menu of contracts offered by the aggregator. According to the revelation principle such an indirect mechanism is equivalent to direct revelation and hence its performance can be analyzed as if the consumers directly revealed the demand function to the aggregator.

Consider the vector $\bar{P}_f \in \mathbb{R}^{N_f}$ containing a finite number of values that $P_f(i)$ may take as an effect of some discretization process assuming for example a uniform grid over the possible values of $P_f(i)$ with $N_f$ discrete points, and construct the $N_f^{N/T}$ different capacity profiles that may occur. For each of them we solve $\mathcal{P}[S_m, \{P_f(i)\}_{i=1}^{N/T}]$ and record $\{\lambda[S_m, P_f(i)]\}_{i=1}^{N/T}$. For each distinct value of $\bar{P}_f(i)$, average among the recorded dual variables that correspond to this capacity value and denote by $\bar{\lambda}[S_m, \bar{P}_f(i)]$ the resulting average dual variable. Since every $T$ steps are decoupled, it suffices to consider here only the $N_f$ profiles for which the capacity limit is constant across the optimization horizon to $\bar{P}_f(i)$, $i = 1, \ldots, N_f$.

The quantity $\bar{\lambda}[S_m, \bar{P}_f(i)]$ is based on the forecast and PV power error values for a given optimization horizon. We can repeat the entire process for different PV power forecasts and error realizations, and then compute the average among all $\bar{\lambda}[S_m, \bar{P}_f(i)]$. With a slight abuse of notation, in the sequel we use the same symbol $\bar{\lambda}[S_m, \bar{P}_f(i)]$ to represent the resulting average quantity. Note that there are two different averaging procedures involved: the first is when constructing $\{\bar{\lambda}[S_m, \bar{P}_f(i)]\}_{i=1}^{N_f}$ from $\{\lambda[S_m, P_f(i)]\}_{i=1}^{N/T}$, and the second is when averaging among the "shadow" prices of problems that correspond to different PV power forecasts. The latter can be thought of as averaging among different representative days in a given time frame.

Therefore, $\bar{\lambda}[S_m, \bar{P}_f(i)]$, corresponds to an average "shadow" price according to which the aggregator will bid for supplying load reduction in the wholesale day-ahead market. Having $\bar{\lambda}[S_m, \bar{P}_f(i)]$ as a function of $\bar{P}_f(i)$, for each $i = 1, \ldots, N_f$ and for the numerical values of Section IV, we can compute the demand bidding curve as shown in Fig. 1 (solid line), which as expected is non-increasing. Notice that, due to the complementarity slackness optimality condition for

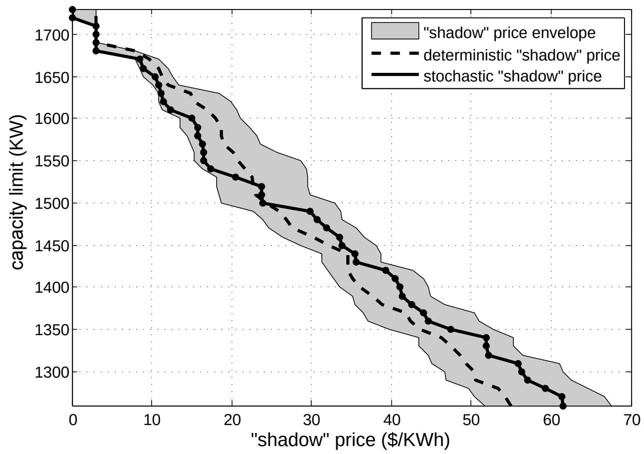


Fig. 1. Demand curve for the stochastic problem (solid line); demand curve for the deterministic problem (dashed line); Demand curve envelope (shaded region), inside which the deterministic and stochastic curves are confined to lie.

$\mathcal{P}[S_m, \{P_f(i)\}_{i=1}^{N/T}]$, having non-zero "shadow" prices implies that the corresponding capacity constraints are binding. This is expected due to the structure of $\mathcal{P}[S_m, \{P_f(i)\}_{i=1}^{N/T}]$.

### *B. Stochastic vs. deterministic "shadow" prices*

The "shadow" price $\bar{\lambda}[S_m, \bar{P}_f(i)]$, $i = 1, \ldots, N_f$, is related to the dual variables associated with the capacity constraints (2). Even though these constraints are deterministic (Proposition 1), the dual variables depend on the uncertainty since (2), (5) are coupled through the decision variables. Therefore, if we consider the deterministic counterpart $\mathcal{P}[\emptyset, \{P_f(i)\}_{i=1}^{N/T}]$ of $\mathcal{P}[S_m, \{P_f(i)\}_{i=1}^{N/T}]$, we get different "shadow" prices $\{\bar{\lambda}[\emptyset, \bar{P}_f(i)]\}_{i=1}^{N_f}$, and hence a different demand curve. This deterministic curve is shown in Fig. 1 (dashed line).

Determining how different the stochastic and the deterministic curves are is not straightforward. We construct an envelope around the deterministic curve, inside which the stochastic one is confined to lie. To this end, let $S_m^+$ be constructed from $S_m$ such that for any sample $\delta_i \in S_m$, $i = 1, \ldots, m$, any element $\delta_i^j(k)$ of $\delta_i$ is replaced by $\max(0, \delta_i^j(k))$. Define $S_m^-$ similarly, with $\delta_i^j(k)$ replaced by $\min(0, \delta_i^j(k))$. That way, $S_m^+$, $S_m^-$ have only non-negative and non-positive elements, respectively. Let $\mathcal{P}[S_m^+, \{P_f(i)\}_{i=1}^{N/T}]$ and $\mathcal{P}[S_m^-, \{P_f(i)\}_{i=1}^{N/T}]$ be the corresponding dispatch problems and $\bar{\lambda}[S_m^+, \bar{P}_f(i)]$, $\bar{\lambda}[S_m^-, \bar{P}_f(i)]$ the associated "shadow" prices computed according to the averaging procedure of the previous subsection, when $S_m$ is substituted with $S_m^+$ and $S_m^-$, respectively.

**Proposition 2.** *For any $m \in \mathbb{N}_+$ and $i = 1, \ldots, N_f$,*

$$1)\ \bar{\lambda}[\emptyset, \bar{P}_f(i)] \in \big[\bar{\lambda}[S_m^+, \bar{P}_f(i)],\ \bar{\lambda}[S_m^-, \bar{P}_f(i)]\big], \quad (7)$$

$$2)\ \bar{\lambda}[S_m, \bar{P}_f(i)] \in \big[\bar{\lambda}[S_m^+, \bar{P}_f(i)],\ \bar{\lambda}[S_m^-, \bar{P}_f(i)]\big]. \quad (8)$$

Proposition 2 shows that the "shadow" price corresponding to (6) for problems in the form of $\mathcal{P}[S_m, \{P_f(i)\}_{i=1}^{N/T}]$ is monotone with respect to the uncertainty error. In particular, if we expect that the uncertain error will only increase ($\delta \in S_m^+$) or will only decrease ($\delta \in S_m^-$), then the demand curve should be shifted towards the left and right, respectively.

If $S_m^+$ or $S_m^-$ is empty, then the corresponding "shadow" price coincides with the one of the deterministic problem. The price envelope is depicted in Fig. 1 and its boundaries correspond to the cases where $\delta \in S_m^+$ and $\delta \in S_m^-$ (if the uncertain error was bounded, the envelope boundaries would correspond to the error extrema). Since "shadow" prices depend on the uncertainty, the computed envelope shows how an uncertain error is translated in the "shadow" price domain. Whether $\bar{\lambda}[S_m, \bar{P}_f(i)]$, $i = 1, \ldots, N/T$, is lower or higher than $\bar{\lambda}[\emptyset, \bar{P}_f(i)]$ can not be decided a priori and depends on the maximum value of $\rho^j(k)$ among the $(j,k)$, $k \in [iT - T + 1, iT]$, indices that correspond to inactive constraints.

A direct consequence of the proof of Proposition 2, is that the "shadow" price in the stochastic set-up will be equal to the "shadow" price of the deterministic one only if the maximum value admitted by $\rho^j(k)$ among the $(j,k)$ indices of the inactive constraints is the same in both problems. This is due to the fact that the dual variable of each capacity constraint is equal to the maximum value attained by the penalty factor $\rho^j(k)$ among all $(j,k)$ indices that correspond to inactive constraints (see proof of Proposition 2). Our analysis depends on the structure of (5), where the uncertainty appears multiplied by the allocation coefficients (see (3),(5)), which all have the same sign due to (4); the validity of these results for other problems needs further investigation.

The economic interpretation is that if the error is expected to be non-negative (similarly for negative error), the total power consumption level is expected to increase, and hence the prices will be lower, as if we had a problem with a capacity limit higher by the amount of the forecast error. Then the expectation about the evolution of prices changes compared to the deterministic case. Therefore, for a given "shadow" price, the consumers are willing to purchase a lower quantity, leading to a shift in the demand curve towards left.

To generalize this statement we quantify the probability with which the computed envelope remains unchanged if a new sample $\delta$ is realized. This can be though of as a probabilistic sensitivity analysis. Consider $\mathcal{P}[S_m \cup \{\delta\}, \{P_f(i)\}_{i=1}^{N/T}]$ and let $\bar{\lambda}[S_m \cup \{\delta\}, \bar{P}_f(i)]$, be the associated "shadow" prices. To simplify the statement of the following theorem, assume that the aforementioned "shadow" prices correspond to the exact dual variables and are not average quantities.

**Theorem 1.** *Assume that $\mathbb{P}$ is any absolutely continuous probability measure. Fix $\epsilon,\ \beta \in (0,1)$. If $m \geq \frac{e}{e-1}\frac{1}{\epsilon}\big(N_{PV}N - 1 + \ln\frac{1}{\beta}\big)$, then for all $i = 1, \ldots, N_f$, with confidence at least $1-\beta$, $\bar{\lambda}[S_m \cup \{\delta\}, \bar{P}_f(i)] \in \big[\bar{\lambda}[S_m^+, \bar{P}_f(i)],\ \bar{\lambda}[S_m^-, \bar{P}_f(i)]\big]$ with probability at least $1-\epsilon$, i.e.*

$$\begin{aligned}\mathbb{P}^m\Big[&(\delta_1, \ldots, \delta_m) \in \Delta^m : \mathbb{P}\Big[\delta \in \Delta : \\ &\bar{\lambda}[S_m \cup \{\delta\}, \bar{P}_f(i)] \in \big[\bar{\lambda}[S_m^+, \bar{P}_f(i)],\ \bar{\lambda}[S_m^-, \bar{P}_f(i)]\big]\Big] \\ &\geq 1-\epsilon\Big] \geq 1-\beta. \end{aligned} \tag{9}$$

$\mathbb{P}^m$ denotes the product probability measure. Theorem 1 offers a probabilistic sensitivity analysis. The interpretation is that for a sufficiently high number of scenarios $m$, with confidence at least $1-\beta$, the "shadow" price $\bar{\lambda}[S_m \cup \{\delta\}, \bar{P}_f(i)]$ that we would obtain by appending one additional uncertainty realization in our scenario set $S_m$ would lie in the envelope $\big[\bar{\lambda}[S_m^+, \bar{P}_f(i)],\ \bar{\lambda}[S_m^-, \bar{P}_f(i)]\big]$ that is constructed using only the $m$ scenarios of $S_m$ with probability at least $1-\epsilon$. In particular, as shown in the proof of Theorem 1, with certain confidence, an additional realization leaves the "shadow" price unchanged, i.e. $\bar{\lambda}[S_m \cup \{\delta\}, \bar{P}_f(i)] = \bar{\lambda}[S_m, \bar{P}_f(i)]$, with probability at least $1-\epsilon$. As $\beta, \epsilon \to 0$ we can claim that, with confidence one, $\bar{\lambda}[S_m \cup \{\delta\}, \bar{P}_f(i)] \in \big[\bar{\lambda}[S_m^+, \bar{P}_f(i)],\ \bar{\lambda}[S_m^-, \bar{P}_f(i)]\big]$ almost surely. On the contrary, if $\beta \to 1$ or $\epsilon \to 1$, then the outer or the inner statement, respectively, in (9) would hold with probability zero, making the conclusion of Theorem 1 trivial and hence of no practical use. Typically, one selects $\epsilon$ and $\beta$ to be close to zero; in particular, $\beta$ can be set to very small values since it appears inside the logarithm in the sample size boubd of Theorem 1 (see also the discussion at the end of Section II-B).

If $\bar{\lambda}[S_m^+, \bar{P}_f(i)],\ \bar{\lambda}[S_m^-, \bar{P}_f(i)]$ were average quantities, computed based on a finite number of "shadow" prices, the result of Theorem 1 would hold with the following modification: Since for every individual "shadow" price that contributes in the average, (9) would be satisfied with possibly different $\epsilon$ and $\beta$, (9) would also hold for the average "shadow" prices with $\epsilon$ and $\beta$ replaced by the sum of the individual $\epsilon,\ \beta$.

### C. Gap between the semi-infinite and the sampled program

The average dual variables $\{\bar{\lambda}[S_m, \bar{P}_f(i)]\}_{i=1}^{N_f}$, have a "shadow" pricing interpretation for $\mathcal{P}[S_m, \{P_f(i)\}_{i=1}^{N/T}]$, however, it is not straightforward how they are related to the dual variables of the semi-infinite program $\mathcal{P}[\Delta, \{P_f(i)\}_{i=1}^{N/T}]$. Let $\mathcal{D}[S_m, \{P_f(i)\}_{i=1}^{N/T}],\ \mathcal{D}[\Delta, \{P_f(i)\}_{i=1}^{N/T}]$ be the dual of the sampled and the semi-infinite program, and denote by $J_{\mathcal{P}_m},\ J_{\mathcal{D}_m}$ and $J_{\mathcal{P}},\ J_{\mathcal{D}}$ the optimal primal and dual objective values of the sampled and semi-infinite program, respectively.

If Slater's condition [34] holds for $\mathcal{P}[\Delta, \{P_f(i)\}_{i=1}^{N/T}]$, then for both the semi-infinite and the sampled program we have zero duality gap. The latter implies that the dual variables of each problem have a "shadow" price interpretation. However, the solution of $\mathcal{D}[S_m, \{P_f(i)\}_{i=1}^{N/T}]$ is not necessarily such that $J_{\mathcal{D}_m} = J_{\mathcal{D}}$, thus implying that the gap between $\mathcal{D}[S_m, \{P_f(i)\}_{i=1}^{N/T}]$ and the semi-infinite problem $\mathcal{P}[\Delta, \{P_f(i)\}_{i=1}^{N/T}]$ might not be zero. Theorem 2 shows that for a sufficiently high number of samples $m$, with certain confidence, the gap between $J_{\mathcal{D}_m}$ and $J_{\mathcal{D}}$ approaches zero as $m \to \infty$ (i.e. $\epsilon \to 0$), and remains bounded by a pre-specified threshold if the number of samples is finite. This supports the fact that we interpret $\{\bar{\lambda}[S_m, \bar{P}_f(i)]\}_{i=1}^{N_f}$ as "shadow" prices.

**Theorem 2.** *Assume that Slater's condition holds for $\mathcal{P}[\Delta, \{P_f(i)\}_{i=1}^{N/T}]$. Fix $\epsilon,\ \beta \in (0,1)$. If $m \geq \frac{e}{e-1}\frac{1}{\epsilon}\big(N_{PV}N - 1 + \ln\frac{1}{\beta}\big)$, then with confidence at least $1-\beta$, $J_{\mathcal{D}} - J_{\mathcal{D}_m} \in [0, I(\epsilon)]$, i.e.*

$$\mathbb{P}^m\Big[(\delta_1, \ldots, \delta_m) \in \Delta^m : J_{\mathcal{D}} - J_{\mathcal{D}_m} \in [0, I(\epsilon)]\Big] \geq 1-\beta, \tag{10}$$

*where $I(\epsilon)$ is such that $\lim_{\epsilon \to 0} I(\epsilon) = 0$ and is given in Theorem 3.6 of [35].*

Note that as $\beta \to 0$ then the gap $J_{\mathcal{D}} - J_{\mathcal{D}_m}$ belongs to $[0, I(\epsilon)]$ almost surely, whereas as $\beta \to 1$ the probability that

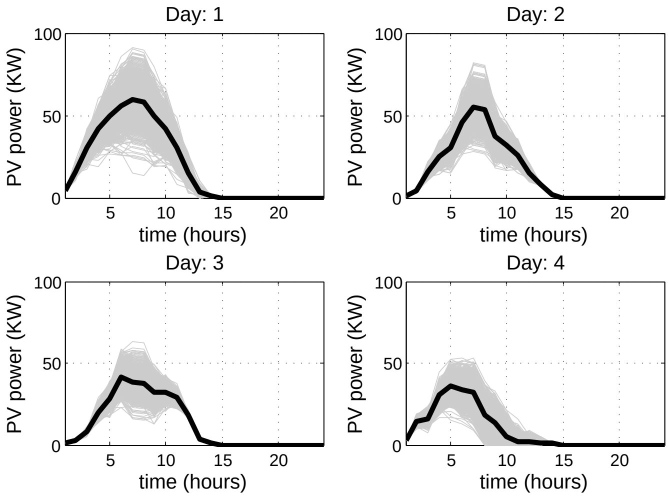


Fig. 2. PV power output for four representative days. For each day, the solid line corresponds to the forecast, whereas the shaded lines show the forecast plus the forecast errors.

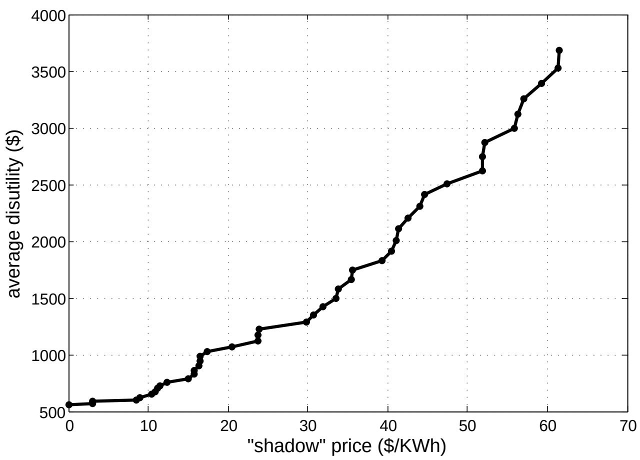


Fig. 3. Average disutility due to load curtailment vs. "shadow" price.

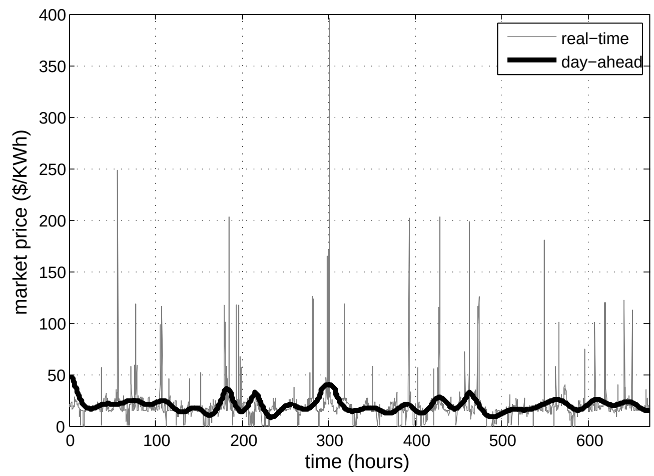


Fig. 4. Day-ahead (solid line) and real-time (shaded line) market price signals for the period 1-28 May 2014.

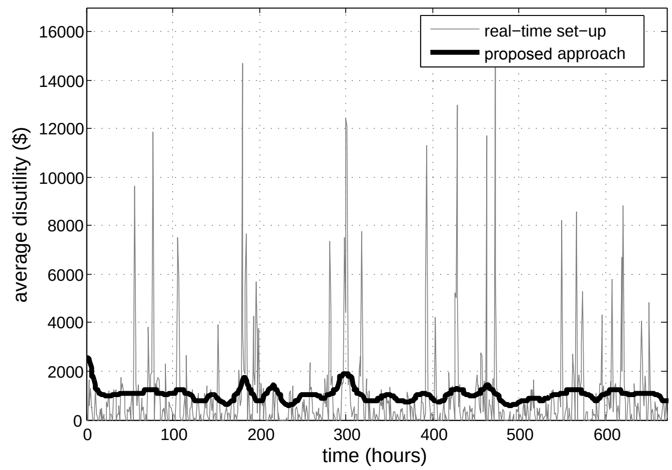


Fig. 5. Average disutility due to load curtailment for the proposed capacity controlled demand side management (solid line) and the real-time (shaded line) price set-up.

$J_{\mathcal{D}} - J_{\mathcal{D}_m} \in [0, I(\epsilon)]$ is zero. For a given confidence $\beta$, as $\epsilon \to 0$ then $I(\epsilon) \to 0$, implying that the gap between the semi-infinite and the sampled program tends to zero, i.e. $J_{\mathcal{D}} = J_{\mathcal{D}_m}$. On the other hand, as $\epsilon \to 1$ then $I(\epsilon) \to \infty$, and the result of Theorem 2 becomes trivial and hence of no practical use.

The proof of Theorem 2 follows from [35] and the duality analysis for semi-infinite programs of [36]. Typically, the dependence of $I(\epsilon)$ on $\epsilon$ is exponential in the number of uncertainty variables $N_{PV}N$, thus increasing rapidly the number of samples that need to be extracted for a given $\epsilon$, $\beta$ to keep $I(\epsilon)$ small enough.

## IV. Simulation Study

We consider the problem described in Section II with $N_L = 3$, $N_c = 5$ and $N_{PV} = 1$. The planning horizon was chosen to be $N = 32$ and we assumed that the capacity limit is communicated every $T = 4$ steps. Every step of the planning horizon corresponds to a 15 minute interval, which implies that the capacity profile has granularity of one hour. Note that our choice for the optimization horizon (i.e. considering the first 8 hours and not the entire day) is motivated by the fact that after that time uncertainty vanishes, so the solution of our formulation approaches the one of the deterministic variant of the problem, and it is not due to computational limitations. The risk metric in (1) was chosen to be the worst case metric based on the first norm. For the sake of this study, we selected $\rho^j(k)$ from a uniform distribution in the interval $[0, 90]$. To compute $\bar{\lambda}[S_m, \bar{P}_f(i)]$, $i = 1, \ldots, N_f$, we selected $N_f = 48$ values starting from 1250KW with granularity of 10KW, and averaged (see discussion in Section III-A) across four representative days in the course of one summer month. For each day the forecast (solid line) and the forecast plus errors (shaded lines) are shown in Fig. 2. The forecast values correspond to normalized data taken from [9]. The PV power output is zero during the hours of no irradiation.

Following [37], to generate forecast error time series for the PV power output, we simulated the stochastic process

$$\delta^j(k+1) = \max\left(\delta^j(k) + u^j(k), -P^j_{PV}(k+1)\right), \qquad (11)$$

with $\delta^j(1) = 0$, until the time-step that corresponds to the peak power production in Fig. 2. After that time, we assumed that the error follows a mirrored pattern so that it degrades until the time of zero production. Variable $u^j(k)$ is extracted from a normal distribution with zero mean and $0.1P^j_{PV}(k)$ standard deviation. In case of multiple PV units, $u^j(k),\ j = 1,\ldots,N_{PV}$, could be extracted from a multivariate normal distribution to take into account spatial correlation as well. The $\max(\cdot,\cdot)$ operator is introduced to ensure that the forecast power plus the generated error does not take negative values. A more involved time-series model, based for example on regression or Markov chains, is outside the scope of this paper. The number of samples we generated was according to Theorem 1, for $\epsilon = 0.03$ and $\beta = 10^{-11}$.

All simulations were carried out using the solver LINPROG (all the resulting scenario based optimization problems are linear programs) under the MATLAB interface YALMIP [38].

### A. Comparison with a real-time market price set-up

We assume that the demand curve constructed in Section III-A is revealed to the aggregator through some mechanism and is used by the aggregator to bid for load reduction in the day-ahead market. For a given day-ahead market price signal (with granularity of one hour), the disutility due to load curtailment (i.e. $\sum_{k=1}^{N}\sum_{j=1}^{N_c}\rho^j(k)\big(P^j_{c,\text{base}}(k) - P^j_c(k)\big)$) for each hour is related to the point on the vertical axis in Fig. 1 that corresponds to this price value along the solid line. This point is a specific capacity limit. Since $\mathcal{P}[S_m, \{P_f(i)\}_{i=1}^{N/T}]$ was parametric with respect to the capacity profile, similarly to the way we computed the average prices $\{\bar{\lambda}[S_m, \bar{P}_f(i)]\}_{i=1}^{N_f}$, we can compute the average disutility that corresponds to each $\bar{P}_f(i),\ i = 1,\ldots,N_f$, and then perform linear interpolation to compute the disutility and any specific capacity limit. Following this procedure we can construct the curve shown in Fig. 3, which shows the average disutility that corresponds to each of the "shadow" prices in Fig. 1. As expected, the higher the "shadow" price, the higher is the disutility due to load curtailment.

To compare the disutility due to load curtailment that occurs when using our capacity control approach, we used as a benchmark a set-up where the loads in the household respond directly to real-time market prices. In other words, the set-point of each device is adjusted dynamically so that the marginal disutility is equal to the real time price. To formulate this real-time price tracking problem, consider the following deterministic optimization program:

$$\mathcal{P}_{RT}: \min_{\left\{\left\{P^j_c(k)\right\}_{j=1}^{N_c}\right\}_{k=1}^{N}} \sum_{k=1}^{N}\sum_{j=1}^{N_c}\Big(\mu(k)P^j_c(k) + U^j(k,0)\Big) \tag{12}$$

subject to

$$\alpha^j(k)P^j_{c,\text{base}}(k) \leq P^j_c(k) \leq P^j_{c,\text{base}}(k),$$
$$\forall j = 1,\ldots,N_c,\ \forall k = 1,\ldots,N. \tag{13}$$

Note that the second term in (12) corresponds to the load disutility evaluated at $\delta = 0$, unlike the objective function of $\mathcal{P}[\Delta, \{P_f(i)\}_{i=1}^{N/T}]$ where a risk metric was employed due to the presence of uncertainty. Constraint (13) is the deterministic variant of (5). Parameter $\mu(k)$ corresponds to the value of the real-time market price signal at time-step $k$. Once problem $\mathcal{P}_{RT}$ is solved, we can compute the disutility $\sum_{j=1}^{N_c} U^j(k,0)$, $k = 1,\ldots,N$, evaluated at the resulting optimal solution. By inspection of $\mathcal{P}_{RT}$, some fraction of load $j$ will be curtailed at time-step $k$ only if $\mu(k) > \rho^j(k)$.

The real-time market price signal used corresponds to normalized data, taken from [39] for the period 1-28 May 2014 with granularity of 15 minutes. The day-ahead market price signal is constructed by averaging accross the real-time one for the same period but for different years; since we are interested in the disutility per hour we also averaged among the intra-hour values to determine an hourly profile. The market price signals are shown in Fig. 4; the solid line indicates the day-ahead market price signal and the shaded line the real-time one.

Fig. 5 shows the resulting disutility due to load curtailment for the proposed approach (solid line) and the real-time price set-up (shaded line). The expected disutility (averaged accross all hours of the price profiles) is $\$1,042.2$ when using capacity controlled and $\$912.8$ for the case where real-time prices are employed, i.e. $14.2\%$ higher disutility. The difference in disutility between the two approaches is a measure for the efficiency loss due to the hierarchical nature of the capacity controlled demand management concept. A moderate difference implies that the proposed approach offers an efficient alternative to real-time price control, without raising stability issues [10], and while offering the consumers the possibility to select a demand response contract (see Section IV-B). Notice that the disutility due to load curtailment follows closely the market price patterns of Fig. 4.

For the rest of this subsection we validate, for a confidence level $\beta = 10^{-11}$, the statement in (9) empirically (without averaging the "shadow" prices; see also discussion below Theorem 1). To achieve this, for a given $S_m$ we performed 10,000 Monte Carlo simulations corresponding to different realizations $\delta \in \Delta$. For each $i = 1,\ldots,N_f$, the empirical probability that $\bar{\lambda}[S_m \cup \{\delta\}, \bar{P}_f(i)] \in \big[\bar{\lambda}[S^+_m, \bar{P}_f(i)],\ \bar{\lambda}[S^-_m, \bar{P}_f(i)]\big]$ can be then computed as the number of simulations out of the 10,000 runs for which the inclusion constraint is satisfied. We found out that this empirical estimate is $0.987$ (it turned out that for this set-up this is the same for all $i = 1,\ldots,N_f$), which is higher compared to the theoretical value $1-\epsilon = 0.97$, implying that the bound in (9) is conservative. The bound in (10) is only of theoretical value and can not be validated numerically, since the semi-infinite problem $\mathcal{P}[\Delta, \{P_f(i)\}_{i=1}^{N/T}]$ is not solvable.

### B. Selection of a service contract

Assume that the aggregator offers a portfolio of a finite number of annual contracts, each of them being a pair of a price ($/KW/year) that the consumer should pay to purchase the contract, and a probability of curtailment as (hours/year). Consider a portfolio with $M$ contracts $\{(c_j,\ p_j)\}_{j=1}^{M}$. To select the most profitable one the consumer can use the information provided by the demand curve.

To achieve this consider a quantization of the demand curve as shown in Fig. 6. All capacity increments have the same width and are centered on the points $\{\bar{P}_f(i)\}_{i=1}^{N_f}$. Inspecting the quantized demand curve, a specific capacity increment corresponds to some $\bar{\lambda}[S_m, \bar{P}_f(i)]$, which shows the change in

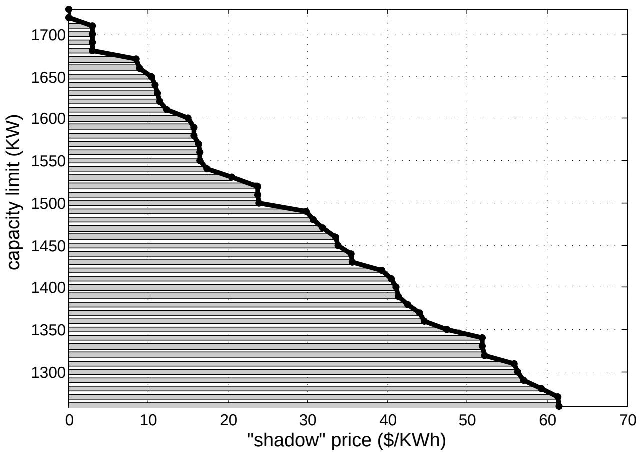


Fig. 6. Quantized demand curve for the stochastic set-up. The solid line is the one of Fig. 1.

disutility due to an incremental change in the capacity limit. The consumer will assign a specific power increment to the contract $i = 1, \dots, M$ that results in the minimum disutility due to load curtailment per KW/year. For each $j = 1, \dots, M$, $i = 1, \dots, N_f$, this is given by $c_j + p_j \bar{\lambda}[S_m, \bar{P}_f(i)]$. Therefore, for a given capacity increment, the index of the most profitable contract in the portfolio is given by

$$j^* = \operatorname{argmin}_j \left(c_j + p_j \bar{\lambda}[S_m, \bar{P}_f(i)]\right), \qquad (14)$$

where it is assumed that there is always a unique contract for which the minimum is achieved. The first term in (14) is the cost of purchasing a contract, whereas the second one represents the cost of load curtailment per KW.

To illustrate this problem we analyze a set-up similar to the one provided in [19]. To this end, assume that the aggregator offers the contract portfolio shown in Table I with $M = 6$. Consider six arbitrarily selected capacity limits. For each of them associate a "shadow" price by means of the demand curve in Fig. 6. These prices are reported in the left column in Table II. Alternatively one could report the associated capacity limits; however, the reason we show the prices is that, together with Table I, they provide the necessary information to calculate the entries of Table II. In fact, these entries correspond to the total disutility per KW/year incurred, i.e. $c_j + p_j \bar{\lambda}[S_m, \bar{P}_f(i)]$, $j = 1, \dots, M$, where $\bar{\lambda}[S_m, \bar{P}_f(i)]$ takes the values shown in the left column of Table II. With boldface we show for each capacity limit the values that correspond to the most profitable contract choice. Using this information, for a given capacity increment (corresponding to some row of Table II), the consumer can select the preferable contract.

TABLE I
CONTRACT PORTFOLIO OFFERED BY THE AGGREGATOR, COMPRISING PAIRS OF PURCHASE PRICES AND PROBABILITIES OF CURTAILMENT.

| Contract $j$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $c_j$ ($/KW/year) | 150 | 120 | 70 | 50 | 30 | 10 |
| $p_j$ (hours/year) | 0.1 | 1 | 5 | 15 | 25 | 50 |

TABLE II
TOTAL DISUTILITY PER KW/YEAR. THE MOST PROFITABLE CONTRACT CORRESPONDS TO THE BOLDFACE ENTRY.

| Price ($/KWh) | Contract | | | | | |
|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 |
| 50 | **155** | 170 | 320 | 800 | 1280 | 2510 |
| 40 | **154** | 160 | 270 | 650 | 1030 | 2010 |
| 30 | 153 | **150** | 220 | 500 | 780 | 1510 |
| 20 | 152 | **140** | 170 | 350 | 530 | 1010 |
| 10 | 151 | 130 | **120** | 200 | 280 | 510 |
| 1 | 150.1 | 121 | 75 | 65 | **55** | 60 |

## V. CONCLUSION

In this paper we considered a capacity control paradigm for demand side management. We formulated the household energy management problem as a stochastic optimization program, quantified the expected disutility due to load curtailment compared to a real-time market price set-up and constructed a probabilistic envelope around the "shadow" prices of the deterministic variant of the problem inside which the "shadow" prices of the stochastic one are confined to lie.

Current work concentrates on enhancing our framework with more involved load dynamics and deferral possibilities. Moreover, since the proposed load management optimization program is parametric with respect to the capacity limits, future work involves selecting them within the optimization process by treating them as decision policies, dependent on the underlying uncertainty. Despite the fact that the pricing analysis presented in this paper was related to load side management, it is also applicable to generation dispatch problems without network constraints. Current focus is to investigate the validity of our pricing analysis in a stochastic, nodal pricing set-up, providing confidence intervals for the locational marginal prices.

## APPENDIX A: INCLUDING STORAGE DEVICES

We enhance the set-up of Section II-A by including $N_s$ storage devices. To model the dynamics of each device we follow the formulation of [26]. Let $x^j(k) \in \mathbb{R}$ be the level of stored energy and $P_s^j(k) \in \mathbb{R}$ be the power exchanged with the storage device $j = 1, \dots, N_s$ at time $k = 1, \dots, N$. The evolution of the energy content of each storage device $j = 1, \dots, N_s$ can be modeled by the following discrete time dynamical system:

$$x^j(k+1) = x^j(k) + \eta^j P_s^j(k) - x_\ell^j, \qquad (15)$$

with $x^j(1) = 0$ (i.e. storage devices start from a zero energy content) for $k = 1, \dots, N-1$. Parameter $x_\ell^j \in \mathbb{R}$ denotes a constant stored energy degradation within each interval of evolution, whereas $\eta^j \in \mathbb{R}$ is the charging/discharging efficiency of each device. Following [40], we use the same efficiency for charging and discharging to simplify the exposition of this section. In case different efficiencies are considered, logical constraints need to be introduced, giving rise to integer variables [26]. The resulting problem would then be a mixed integer optimization program; the analysis of Section III would still be applicable with a few modifications. In particular, the number of integer variables would appear additive to $N_{PV}N$ in the sample size bounds of Theorems 1 and 2 (see also [35]).

The household energy management problem of Section II-A would then involve minimizing (1) with respect to the decision variables of $\mathcal{P}[\Delta, \{P_f(i)\}_{i=1}^{N/T}]$ and $\{\{P_s^j(k)\}_{j=1}^{N_s}\}_{k=1}^{N}$, subject to (2)-(5), the storage dynamics (15) and possible limits on the energy of each device $\underline{x}^j \leq x^j(k) \leq \overline{x}^j$, for $j = 1, \ldots, N_s$, $k = 1, \ldots, N$. The only modification is that the term $\sum_{j=1}^{N_s} P_s^j(k)$ should be added inside the outer summation in the left-hand side of (2).

Even though introducing storage dynamics in the load energy management problem is straightforward, it poses challenges when following the averaging procedure of Section III-A. The reason is that now every consecutive $T$ steps in the optimization problem are coupled due to (15), therefore to compute a demand curve $N_f^{N/T}$ different capacity profiles need to be constructed and for each of them the optimization problem outlined above has to be solved. In principle, if sufficient resources are available these problems can be solved in parallel and be allocated to different processors. In this paper, however, to get an estimate of the disutility due to load curtailment we introduce the simplifying assumption that the capacity limit remains constant over the entire optimization horizon. This implies that we only need to consider $N_f$ capacity profiles, one for each value of $\bar{P}_f(i)$.

We revisited the study of Section IV-A including $N_s = 1$ storage devices, and compared the capacity control approach with the real-time price set-up in terms of disutility due to load curtailment. The proposed approach resulted in $19.1\%$ higher disutility compared to the case where curtailment occurs in response to real-time prices. Including storage devices offers additional degrees freedom, hence the optimal objective value of the associated problem (and also the one of the dual problem due to strong duality) is lower compared to the one where no storage dynamics are considered. This is not necessarily the case, however, with the resulting disutility calculated according to the demand curve construction of Section III-A, which may be higher compared to the case where no storage devices are included. This is due to the averaging procedure of Section III-A, where we average among "shadow" prices that correspond to the capacity constraints of every consecutive $T$ steps. These consecutive $T$ steps are coupled via the storage dynamics leading to an objective for the dual problem that involves a weighted sum of the "shadow" prices that correspond to the capacity constraint of every $T$ steps. To compute the average quantities, however, we use the sum of the "shadow" prices corresponding to the consecutive $T$ steps, which, unlike the optimal dual objective value (it includes a weighted sum of the "shadow" prices instead), is not necessarily lower compared to the case where no storage is included. If we did not perform this averaging step the resulting disutility would always be lower in the case where storage dynamics are considered.

## Appendix B: Proofs

**Proof of Proposition 1.** The proof follows from the analysis of [6]. It suffices to show that (6) emanates from (2), (4). Since the last two terms in (3) are mutually exclusive, assume that the first one is nonzero. In the opposite case the proof is analogous. Substituting (3) in (2), for $i = 1, \ldots, N/T$,

$$\sum_{k=iT-T+1}^{iT} \Big[ \sum_{j=1}^{N_L} P_L^j(k) - \sum_{j=1}^{N_{PV}} \big( P_{PV}^j(k) + \delta^j(k) \big) + \sum_{j=1}^{N_c} P_c^j(k) + \sum_{j=1}^{N_c} d_+^j(k) \sum_{\ell=1}^{N_{PV}} \delta^\ell(k) \Big] \leq P_f(i),\ \forall \delta \in \Delta, \tag{16}$$

Since $\sum_{j=1}^{N_c} d_+^j(k) = 1$ due to (4), the terms that involve the uncertainty in (16) cancel out. The resulting inequality is the one given in (6), concluding the proof. □

**Proof of Proposition 2.** 1) We only show that $\bar{\lambda}[S_m^+, \bar{P}_f(i)] \leq \bar{\lambda}[\emptyset, \bar{P}_f(i)]$; the proof that $\bar{\lambda}[\emptyset, \bar{P}_f(i)] \leq \bar{\lambda}[S_m^-, \bar{P}_f(i)]$ follows symmetric arguments. The deterministic variant of (5), which affects $\bar{\lambda}[\emptyset, \bar{P}_f(i)]$, would be

$$\alpha^j(k) P_{c,\text{base}}^j(k) \leq P_c^j(k) \leq P_{c,\text{base}}^j(k). \tag{17}$$

If we consider $\mathcal{P}[S_m^+, \{P_f(i)\}_{i=1}^{N/T}]$, (5) is enforced for all $\delta \in S_m^+$. By rearranging some terms, for all $\delta \in S_m^+$ we have that

$$\alpha^j(k) P_{c,\text{base}}^j(k) \leq P_c^j(k) \leq P_{c,\text{base}}^j(k) - d_+^j(k) \sum_{\ell=1}^{N_{PV}} \delta^\ell(k). \tag{18}$$

Since any feasible $d_+^j(k)$ is non-negative and $\sum_{\ell=1}^{N_{PV}} \delta^\ell(k) \geq 0$, (18) implies that for all $k = 1, \ldots, N$, and any $\delta \in S_m^+$, the upper bound of $P_c^j(k)$ can only be tighter compared to (17).

Notice that, either if we use the expected or the worst-case value, $\mathcal{R}_{\delta \in S_m^+}[\cdot]$ is such that every term in (1) is of the form $\rho^j(k)\big(P_{c,\text{base}}^j(k) - P_c^j(k)\big) - \rho^j(k)\big(\mathcal{R}_{\delta \in S_m^+}[d_+^j(k) \sum_{\ell=1}^{N_{PV}} \delta^\ell(k)]\big)$. We show that, for any $\delta \in S_m^+$, if a constraint (in the sense of $(j,k)$ indices) that at the optimal solution of $\mathcal{P}[\emptyset, \{P_f(i)\}_{i=1}^{N/T}]$ was active/binding at the upper limit of (17), then it will remain active at the upper limit of (18) at the optimal solution of $\mathcal{P}[S_m^+, \{P_f(i)\}_{i=1}^{N/T}]$. Note that $\mathcal{P}[S_m^+, \{P_f(i)\}_{i=1}^{N/T}]$ is feasible due to the assumption stated at the end of Section II-B.

To see this, assume for the sake of contradiction that at the optimal solution of $\mathcal{P}[S_m^+, \{P_f(i)\}_{i=1}^{N/T}]$, some of the constraints that at the optimal solution of $\mathcal{P}[\emptyset, \{P_f(i)\}_{i=1}^{N/T}]$ were active at the upper limit, become inactive or are active at the lower limit. Here we assume that the set of constraints that are active at the upper limit of $\mathcal{P}[\emptyset, \{P_f(i)\}_{i=1}^{N/T}]$ is non-empty. In the opposite case, the proof is analogous. The $(j,k)$ indices of the constraints that are active at the upper limit for $\mathcal{P}[\emptyset, \{P_f(i)\}_{i=1}^{N/T}]$ correspond to the highest $\rho^j(k)$; otherwise the corresponding terms $\rho^j(k)\big(P_{c,\text{base}}^j(k) - P_c^j(k)\big)$ would be strictly positive, thus increasing the objective value. Therefore, if some of these constraints become inactive for $\mathcal{P}[S_m^+, \{P_f(i)\}_{i=1}^{N/T}]$, then for any $d_+^j(k)$, and hence also for the optimal one, $\sum_{k=1}^{N} \sum_{j=1}^{N_c} \rho^j(k)\big(P_{c,\text{base}}^j(k) - P_c^j(k)\big)$ would be higher compared to a solution where all of them remain active at the upper limit. Consider a new solution using the same $\{\{d_+^j(k)\}_{j=1}^{N_c}\}_{k=1}^{N}$, and by appropriately adjusting $P_c^j(k)$ so that all constraints that were active at the upper limit remain active. Due to the fact that the capacity limits in

(6) are "budget" type of constraints, the constructed solution is ensured to be feasible. Since we used the same allocation coefficients, $\rho^j(k)\mathcal{R}_{\delta\in S_m^+}[d_+^j(k)\sum_{\ell=1}^{N_{PV}}\delta^\ell(k)]$ is the same for both solutions, implying that the new solution can only have a lower objective value, thus violating the optimality hypothesis and establishing a contradiction.

Therefore, all constraints that were active at the upper limit for $\mathcal{P}[\emptyset,\{P_f(i)\}_{i=1}^{N/T}]$, remain active for $\mathcal{P}[S_m^+,\{P_f(i)\}_{i=1}^{N/T}]$. Hence, the number of constraints that at the optimal solution of $\mathcal{P}[S_m^+,\{P_f(i)\}_{i=1}^{N/T}]$ are active at the upper limit, is higher than or equal to the one of $\mathcal{P}[\emptyset,\{P_f(i)\}_{i=1}^{N/T}]$. Moreover, following an analysis similar to [41], we can show via the Karush-Kuhn-Tucker (KKT) optimality conditions that, for each $i=1,\dots,N/T$, the dual variable associated with (2) is equal to the penalty coefficient $\rho^j(k)$, with $(j,k)$, $k\in[iT-T+1,iT]$, being the indices of the *inactive* constraint with the highest $\rho^j(k)$. This is similar to the fact that, in case of a linear objective function and for an uncongested, lossless network, the locational marginal price is determined by the marginal generator with the lowest cost [41]. The difference is due to the fact that, in contrast to the generating units in [41], $P_c^j(k)$ appears with a negative sign in the objective function.

Since it was shown that more constraints may be active at the upper limit in $\mathcal{P}[S_m^+,\{P_f(i)\}_{i=1}^{N/T}]$, and the ones that are active are always those with the highest $\rho^j(k)$, the value of $\rho^j(k)$ and hence the dual variable of (2) in $\mathcal{P}[S_m^+,\{P_f(i)\}_{i=1}^{N/T}]$ can not exceed the one of $\mathcal{P}[\emptyset,\{P_f(i)\}_{i=1}^{N/T}]$. Since $\bar{\lambda}[S_m^+,\bar{P}_f(i)]$, $\bar{\lambda}[\emptyset,\bar{P}_f(i)]$ are average values of these quantities, the last statement implies that $\bar{\lambda}[S_m^+,\bar{P}_f(i)]\leq\bar{\lambda}[\emptyset,\bar{P}_f(i)]$ and concludes the first part of the proof.

2) Consider $\mathcal{P}[S_m,\{P_f(i)\}_{i=1}^{N/T}]$. For each $\delta\in S_m$, $\sum_{\ell=1}^{N_{PV}}\delta^\ell(k)$, may be either positive or negative, since not necessarily all error samples have the same sign. For each $k=1,\dots,N$, consider the sign of $\sum_{\ell=1}^{N_{PV}}\delta^\ell(k)$ for the sample $\delta\in S_m$ for which it achieves its maximum absolute value, i.e. the sign of the worst-case error per time-step. In fact, it suffices to enforce the constraints only for these worst-case errors and not for all samples. If this sign is positive then more constraints may be active at the upper limit of $P_c^j(k)$ compared to $\mathcal{P}[\emptyset,\{P_f(i)\}_{i=1}^{N/T}]$, otherwise fewer constraints may be active.

In any case, the maximum absolute value of $\sum_{\ell=1}^{N_{PV}}\delta^\ell(k)$ can only be lower (since the individual samples may have opposite signs) from the case where $\delta\in S_m^+$ or $\delta\in S_m^-$. Therefore, in $\mathcal{P}[S_m,\{P_f(i)\}_{i=1}^{N/T}]$ we tighten either the upper or the lower constraint limit compared to $\mathcal{P}[\emptyset,\{P_f(i)\}_{i=1}^{N/T}]$ but by a smaller amount compared to the cases where $\delta\in S_m^+$ and $\delta\in S_m^-$, respectively. From the first part of the proof we then have that $\bar{\lambda}[S_m,\bar{P}_f(i)]\in\big[\bar{\lambda}[S_m^+,\bar{P}_f(i)],\ \bar{\lambda}[S_m^-,\bar{P}_f(i)]\big]$, thus concluding the second part of the proof. □

**Proof of Theorem 1.** By [30], if $m\geq\frac{e}{e-1}\frac{1}{\epsilon}\big(N_{PV}N-1+\ln\frac{1}{\beta}\big)$, then for all $i=1,\dots,N_f$, with confidence at least $1-\beta$, a new sample $\delta\in\Delta$ would give rise to a constraint, for which the optimal solution of $\mathcal{P}[S_m,\{P_f(i)\}_{i=1}^{N/T}]$ would be strictly feasible with probability at least $1-\epsilon$. Strict feasibility is ensured since, due to the continuity assumption for $\mathbb{P}$, if the constraints that are affine with respect to the uncertainty are active, they would form a lower dimensional manifold in the uncertainty space. The latter implies that for any $S_m$, the probability of $\delta\in\Delta$ belonging to this manifold, thus giving rise to an active constraint, is of measure zero.

Due to the complementarity slackness condition [34], with confidence at least $1-\beta$, the dual variable associated with the new constraint would be zero with probability at least $1-\epsilon$ and hence the dual variable associated with the capacity constraint remains unaffected, i.e. $\bar{\lambda}[S_m\cup\{\delta\},\bar{P}_f(i)]=\bar{\lambda}[S_m,\bar{P}_f(i)]$ (it was assumed that these are not average quantities). The last statement can be equivalently written as

$$\begin{aligned}&\mathbb{P}^m\Big[(\delta_1,\dots,\delta_m)\in\Delta^m:\\&\mathbb{P}\Big[\delta\in\Delta:\ \bar{\lambda}[S_m\cup\{\delta\},\bar{P}_f(i)]=\bar{\lambda}[S_m,\bar{P}_f(i)]\Big]\geq1-\epsilon\Big]\\&\geq1-\beta.\end{aligned}\tag{19}$$

By Proposition 2, for any $S_m\subset\Delta$, $\bar{\lambda}[S_m,\bar{P}_f(i)]\in\big[\bar{\lambda}[S_m^+,\bar{P}_f(i)],\ \bar{\lambda}[S_m^-,\bar{P}_f(i)]\big]$. The last statement, together with (19), leads to (9) and hence concludes the proof. □

**Proof of Theorem 2.** $\mathcal{D}[\Delta,\{P_f(i)\}_{i=1}^{N/T}]$ is an infinite dimensional optimization program (it involves optimizing with respect to all positive valued functions of the uncertainty) [36], as the dual of a semi-infinite one. On the other hand, for any $S_m\subset\Delta$, $\mathcal{D}[S_m,\{P_f(i)\}_{i=1}^{N/T}]$ has a finite number of dual variables, corresponding to the samples in $S_m$. We can then construct a function of the uncertainty as the weighted sum of dirac functions located at the samples with weight coefficients equal to the value of the dual variables (see also [36]). Clearly the constructed function will be a suboptimal solution for $\mathcal{D}[\Delta,\{P_f(i)\}_{i=1}^{N/T}]$, since the latter involves optimizing over positive valued functions with a generic dependence on the uncertainty. Therefore, for any $S_m\subset\Delta$, $J_{\mathcal{D}_m}\leq J_{\mathcal{D}}$.

Under Slater's condition [34] there is zero duality gap between $\mathcal{P}[\Delta,\{P_f(i)\}_{i=1}^{N/T}]$ and $\mathcal{D}[\Delta,\{P_f(i)\}_{i=1}^{N/T}]$, and hence also between the scenario based programs $\mathcal{P}[S_m,\{P_f(i)\}_{i=1}^{N/T}]$ and $\mathcal{D}[S_m,\{P_f(i)\}_{i=1}^{N/T}]$ [36]. The latter implies that for all $S_m\subset\Delta$, $J_{\mathcal{P}}=J_{\mathcal{D}}$ and $J_{\mathcal{P}_m}=J_{\mathcal{D}_m}$. Therefore,

$$J_{\mathcal{P}_m}=J_{\mathcal{D}_m}\leq J_{\mathcal{D}}=J_{\mathcal{P}}.\tag{20}$$

Moreover, under Slater's condition and the feasibility assumption for $\mathcal{P}[S_m,\{P_f(i)\}_{i=1}^{N/T}]$ (see discussion at the end of Section II-B), it is shown in Theorem 3.6 of [35] that

$$\mathbb{P}^m\Big[(\delta_1,\dots,\delta_m)\in\Delta^m:J_{\mathcal{P}}-J_{\mathcal{P}_m}\in[0,I(\epsilon)]\Big]\geq1-\beta,\tag{21}$$

for some function $I(\epsilon)$ such that $\lim_{\epsilon\to0}I(\epsilon)=0$. Statements (20) and (21) lead to (10) and conclude the proof. □

## References


[1] L. Xie, P. M. S. Carvalho, L. A. F. M. Ferreira, J. Liu, B. H. Krogh, N. Popli, and M. D. Ilic, "Wind Integration in Power Systems: Operational Challenges and Possible Solutions," *Proceedings of the IEEE*, vol. 99, no. 1, pp. 214 – 232, 2011.

[2] F. D. Galiana, F. Bouffard, J. M. Arroyo, and J. F. Restrepo, "Scheduling and pricing of coupled energy and primary, secondary, and tertiary reserves," *Proceeding of the IEEE*, vol. 93, no. 11, pp. 1970–1983, 2005.

[3] F. Bouffard and F. D. Galiana, "Stochastic security for operations planning with significant wind power generation," *IEEE Transactions on Power Systems*, vol. 23, no. 2, pp. 306 – 316, 2008.
[4] J. M. Morales, A. J. Conejo, and J. Perez-Ruiz, "Economic valuation of reserves in power systems with high penetration of wind power," *IEEE Transactions on Power Systems*, vol. 24, no. 2, pp. 900–910, 2009.
[5] A. Papavasiliou, S. S. Oren, and R. P. O'Neill, "Reserve requirements for wind power integration: A scenario-based stochastic programming framework," *IEEE Transactions on Power Systems*, vol. 26, no. 4, pp. 2197 – 2206, 2011.
[6] M. Vrakopoulou, K. Margellos, J. Lygeros, and G. Andersson, "A Probabilistic Framework for Reserve Scheduling and N-1 Security Assessment of Systems With High Wind Power Penetration," *IEEE Transactions on Power Systems*, vol. 28, no. 4, pp. 3885–3896, 2013.
[7] G. Strbac, "Demand side management: Benefits and challenges," *Energy Policy*, vol. 36, no. 12, pp. 4419 – 4426, 2008.
[8] D. S. Callaway and I. A. Hiskens, "Achieving Controllability of Electric Loads," *Proceedings of the IEEE*, vol. 99, no. 1, pp. 184 – 199, 2011.
[9] E. Vrettos, K. Lai, F. Oldewurtel, and G. Andersson, "Predictive Control of Buildings for Demand Response with Dynamic Day-ahead and Real-time Prices," *European Control Conference*, pp. 2527–2534, 2013.
[10] M. Roozbehani, M. A. Dahleh, and S. K. Mitter, "Volatility of power grids under real-time pricing," *IEEE Transactions on Power Systems*, vol. 27, no. 4, p. 19261940, 2012.
[11] D. S. Callaway, "Tapping the energy storage potential in electric loads to deliver load following and regulation, with application to wind energy," *Energy Conversion and Management*, vol. 50, no. 5, pp. 1389 – 1400, 2009.
[12] J. L. Mathiew, S. Koch, and D. S. Callaway, "State Estimation and Control of Electric Loads to Manage Real-Time Energy Imbalance," *IEEE Transactions on Power Systems*, vol. 28, no. 1, pp. 430 – 440, 2013.
[13] Z. Ma, D. S. Callaway, and I. A. Hiskens, "Decentralized charging control for large populations of plug-in electric vehicles," *IEEE Transactions on Control Systems Technology*, vol. 21, no. 1, pp. 69 – 78, 2013.
[14] M. Vrakopoulou, J. L. Mathieu, and G. Andersson, "Stochastic Optimal Power Flow with Uncertain Reserves from Demand Response," *Hawaii International Conference on System Sciences*, pp. 2353 – 2362, 2014.
[15] M. Kamgarpour, C. Ellen, S. E. Z. Soudjani, S. Gerwinn, J. L. Mathieu, N. Mullner, A. Abate, D. S. Callaway, M. Fraenzle, and J. Lygeros, "Modeling options for demand side participation of thermostatically controlled loads ," *Bulk Power System Dynamics and Control - IX Optimization, Security and Control of the Emerging Power Grid (IREP), 2013 IREP Symposium*, pp. 1–15, 2013.
[16] N. Lu, "An Evaluation of the HVAC Load Potential for Providing Load Balancing Service," *IEEE Transactions on Smart Grid*, vol. 3, no. 3, pp. 1263–1270, 2012.
[17] N. Lu and Y. Zhang, "Design Considerations of a Centralized Load Controller Using Thermostatically Controlled Appliances for Continuous Regulation Reserves," *IEEE Transactions on Smart Grid*, vol. 4, no. 2, pp. 914–921, 2013.
[18] Y. Zhang and N. Lu, "Parameter Selection for a Centralized Thermostatically Controlled Appliances Load Controller Used for Intra-Hour Load Balancing," *IEEE Transactions on Smart Grid*, vol. 4, no. 4, pp. 2100–2108, 2013.
[19] S. S. Oren, "A Historical Perspective and Business Model for Load Response Aggregation Based on Priority Service," *Hawaii International Conference on System Sciences*, pp. 1–9, 2013.
[20] A. Nayyar, J. Taylor, A. Subramanian, K. Poolla, and P. Varaiya, "Aggregate flexibility of a collection of loads," *IEEE Conference on Decision and Control*, pp. 5600–5607, 2013.
[21] G. L. Doorman, "Capacity Subscription: Solving the Peak Demand Challenge in Electricity Markets," *IEEE Transactions on Power Systems*, vol. 20, no. 1, pp. 239–245, 2005.
[22] H. Chao, S. S. Oren, S. A. Smith, and R. B. Wilson, "Multilevel Demand Subscription Pricing for Electric Power," *Energy Economics*, vol. 8, no. 4, pp. 199–217, 1986.
[23] S. S. Oren, S. A. Smith, and R. B. Wilson, "Multi-product Pricing for Electric Power," *Energy Economics*, vol. 9, no. 2, pp. 104–114, 1987.
[24] H. Chao, S. S. Oren, S. A. Smith, and R. B. Wilson, "Priority Service: Market Structure and Competition," *The Energy Journal, Special Issue on Electricity Reliability*, vol. 9, no. 2, pp. 77–104, 1988.
[25] T. Strauss and S. S. Oren, "Priority Pricing of Interruptible Electric Power With an Early Notification Option," *The Energy Journal*, vol. 14, no. 2, pp. 175–195, 1993.
[26] A. Parisio, E. Rikos, and L. Glielmo, "A Model Predictive Control Approach to Microgrid Operation Optimization," *IEEE Transactions on Control Systems Technology*, vol. 22, no. 5, pp. 1813–1827, 2014.
[27] P. Stluka, D. Godbole, and T. Samad, "Energy management for buildings and microgrids," *IEEE Conference on Decision and Control*, p. 51505157, 2011.
[28] R. R. Negenborn, M. Houwing, B. D. Schutter, and J. Hellendoom, "Model predictive control for residential energy resources using a mixed-logical dynamic model," *IEEE International Conference on Networking, Sensing and Control*, p. 702707, 2009.
[29] S. Maharjan, Q. Zhu, Y. Zhang, S. Gjessing, and T. Basar, "Dependable Demand Response Management in the Smart Grid: A Stackelberg Game Approach," *IEEE Transactions on Smart Grid*, vol. 4, no. 1, pp. 120–132, 2013.
[30] X. Zhang, S. Grammatico, G. Schildbach, P. Goulart, and J. Lygeros, "On the Sample Size of Randomized MPC for Chance Constrained Systems with Application to Building Climate Control," *European Control Conference*, pp. 478–483, 2014.
[31] G. C. Calafiore and M. C. Campi, "The scenario approach to robust control design," *IEEE Transactions on Automatic Control*, vol. 51, no. 5, pp. 742 – 753, 2006.
[32] K. Margellos, M. Prandini, and J. Lygeros, "On the connection between compression learning and scenario based optimization," *IEEE Transactions on Automatic Control, conditionally accepted*, 2014. [Online]. Available: http://arxiv.org/abs/1403.0950
[33] J. J. Laffont and D. Martimort, "The Theory of Incentives: The Principal-Agent Model," *Princeton University Press*, 2001.
[34] D. Bertsekas, *Convex Optimization Theory*. Athena Scientific, 2009.
[35] P. Mohajerin, T. Sutter, and J. Lygeros, "Performance bounds for the scenario approach and an extension to a class of non-convex programs," *IEEE Transactions on Automatic Control*, vol. 60, no. 1, pp. 46–58, 2015.
[36] A. Shapiro, "Semi-infinite programming, duality, discretization and optimality conditions," *Optimization*, vol. 58, no. 2, pp. 133–161, 2009.
[37] J. Warrington, P. Goulart, S. Mariethoz, and M. Morari, "Policy-based reserves for power systems," *IEEE Transactions on Power Systems*, vol. 28, no. 4, pp. 4427–4437, 2013.
[38] J. Löfberg, "YALMIP: A toolbox for modeling and optimization in MATLAB," *IEEE International Symposium on Computer Aided Control Systems Design*, pp. 284–289, 2005.
[39] California ISO, "Settling the Day-Ahead Market," *Technical report*, 2009. [Online]. Available: http://www.energyonline.com/Data/GenericData.aspx?DataId=20#
[40] E. Vrettos, A. Witzig, R. Kurmann, S. Koch, and G. Andersson, "Maximizing Local PV Utilization Using Small-Scale Batteries and Flexible Thermal Loads," *European PV Solar Energy Conference and Exhibition*, pp. 1–13, 2013.
[41] T. Orfanogianni and G. Gross, "A General Formulation for LMP Evaluation," *IEEE Transactions on Power Systems*, vol. 22, no. 3, pp. 1163–1173, 2007.

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